# Final Exam Cosmology - Study year 2024-2025

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The exam duration is 2 hours.
The grade is out of 100. Please answer all the questions in the exam.
No external material is allowed, except the formula sheet provided to you in the exam and a calculator.
The number of points given to each question is indicated next to it. Indicate clearly the steps in your solution and provide sufficient text.
**If you are unable to answer a subquestion on which a later question depends, please answer the later question by outlining the method clearly without plugging in the number from the previous one.**

Q I. This question is composed of 5 short questions with 10 points each.

1. The mass of the Milky way is about $10^{12} M_\odot$ (where $M_\odot \approx 2 \times 10^{30}$ kg). Given the current mass density of the Universe what is the radius from which the matter that formed the Milky way have accumulated? Hint: Assume that the Milky way has formed from a homogeneous density distribution.
2. In the Steady State model the Universe expands exponentially like a dark energy dominated Universe (which is what we think our Universe currently is). Despite this the Steady State theory is ruled out. Give two reasons for why the Steady State model is not a valid model.
3. Explain why we need 60-efolds expansion during inflation.
4. What are the two time scales involved in the Jeans Instability and how they determine the Jeans mass?
   Consider the case of a mixture of dark matter and baryons, how this will change the baryonic Jeans mass?
5. On the face of it, recombination should happen when the temperature of the Universe corresponds to $13.6\,eV$,the ionization energy of hydrogen, namely at temperature of about 15780 Kelvin ($z = 57800$). Instead, recombination happened at $z = 1100$. Explain why does recombination is delayed for such a long time?

Q II. In the class we discussed Friedmann's equation for a Universe with 3 components, matter, radiation and Dark Energy (DE), together with curvature. We assumed for the DE component an equation of state parameter $w = -1$. However, for this question we consider the case in which the equation of state parameter is not necessary $-1$, but rather a generic value, $w$. Assuming all the other components remain the same as in the class, please answer the following questions. Express your answer in terms of $\Omega_{m,0}$, $\Omega_{r,0}$, $\Omega_{DE,0}$, $\Omega_{\kappa,0}$, and $H_0$.

1. **Write the generic form of Friedmann's eq. in this case** (7 points)
2. **What is the maximum value (the least negative) of $w$, so that any Universe will still be dominated by the DE component in the future (as $a \to \infty$), and why?** (5 points)

Now assume a Universe that contains neither matter, radiation nor curvature components. Assume further that only component in this Universe, i.e., the DE component, has an equation of state parameter $w = -\frac{2}{3}$. Obviously, the current energy DE density in terms of the critical density in this case is $\Omega_{DE,0} = 1$. Assume that you know the current value of Hubble constant, $H_0$.

3. **What is the scale factor as a function of time, $a(t)$?** (10 points)
4. **What is the age of the Universe in this case? Express your answer in terms of $H_0$.** (3 points)
5. **What is the comoving distance as a function of redshift, $\chi(z)$, in this case? (See eq. in the Formulae Sheet)** (8 points)

For the next part of the question, assume a Universe that contains only DE component with $w = -\frac{2}{3}$, and curvature component (without matter and radiation). Assume further that the current values of the current DE and curvature in terms of the critical density is $\Omega_{DE,0}$ and $\Omega_{\kappa,0}\,(= 1 - \Omega_{DE,0})$, and Hubble constant is $H_0$.

6. **Write and solve Friedmann's equation in this case to find the scale factor as a function of time.** (12 points)
7. **Based on the answer to the previous subquestion, at what time does the contribution of the DE component to the value of the scale factor,$a$, starts to dominate over that of the curvature component in such a Universe?** (5 points)

Total number of points for this question is 50

**Good Luck!**